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A note on Gibbard's proof

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Abstract A proof by Allan Gibbard (Ifs: Conditionals, beliefs, decision, chance, time. Reidel, Dordrecht, 1981) seems to demonstrate that if indicative conditionals have truth conditions, they cannot be stronger than material implication. Angelika Kratzer's theory that conditionals do not denote two-place operators purports to escape this result [see Kratzer (Chic Linguist Soc 22(2):1–15, 1986, 2012)]. In this note, I raise some trouble for Kratzer's proposed method of escape and then show that her semantics avoids this consequence of Gibbard's proof by denying modus ponens. I also show that the same holds for Anthony Gillies' semantics (Philos Rev 118(3):325–349, 2009) and argue that this consequence of these theories is not obviously prohibitive—hence, both remain viable theories of indicative conditionals.

Keywords Philosophy of language · Indicative conditionals · Gibbard's proof · Modus ponens

Consider the following indicative conditional:

(1) If a Republican won the election, Reagan did.

It's controversial whether sentences like (1) express propositions, in part because of a proof devised by Gibbard (1981) that seems to establish:

(2) Gibbard's conclusion:
An indicative conditional *if p, q* is either equivalent to the material conditional $p \supset q$ or does not express a proposition.

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Although many theorists accept Gibbard's conclusion,[1] there are reasons for rejecting it. Intuitively, (1) is stronger than material implication—that is, the falsity of its antecedent is not sufficient for its truth[2]—and indicatives behave otherwise as if they express propositions (they embed under attitude verbs and in the scopes of quantificational determiners, for example). So, is there any way of avoiding Gibbard's conclusion in light of Gibbard's proof?

Gibbard's proof relies on the assumption that indicative conditionals denote two-place conditional operators the interpretation of which is invariant across linguistic contexts—hence, we might try to avoid Gibbard's conclusion by rejecting this assumption. This is just what the theories of Kratzer (1986, 2012) and Gillies (2009, 2010) aim to do. Kratzer proposes that conditionals do not denote two-place operators at all and Gillies proposes that conditionals denote a conditional operator that is both index- and context-shifty, and hence potentially has a different interpretation when embedded within the scope of another conditional operator than when unembedded.

In this note, I argue that things are not so simple. Focusing primarily on Kratzer's proposal, I argue that simply denying the assumption that indicative conditionals denote two-place conditional operators is not enough to avoid Gibbard's conclusion, since we can state the proof without that assumption. Nonetheless, I go on to show that both Kratzer and Gillies' theories in fact avoid Gibbard's conclusion, but they do so by invalidating modus ponens—this seems to be the cost for stronger-than-material-conditional propositionalism about indicative conditionals [cf. McGee (1985)]. I argue that this cost isn't obviously prohibitive, and therefore that both Kratzer's restrictor theory and Gillies' doubly-shifty theory remain viable propositionalist theories of indicative conditionals.

[1] Material conditional theorists include (Grice 1989; Lewis 1976; Jackson 1979). Non-propositionalists include (Adams 1975; Gibbard 1981; Edgington 1995; Bennett 2003).

[2] For instance, though it has a false antecedent, the following conditional seems false:

(i) If the moon is made of cheese, there are several thousand moon-rats eating away at it.

The standard defense of the material conditional theory holds that (i) is unassertable rather than false. I'm doubtful such defenses will actually succeed, but even if they do, there's an often overlooked problem for the material conditional theory that doesn't seem defensible in the same way [pointed out by Grice (1989, p. 85)]. The problem is that according to such a theory, the negation of *if p, q* entails the truth of *p* and the falsity of *q*, but that's not correct:

(ii) I don't know whether a Republican won the election, but of course, since Anderson didn't win, I know that it's not the case that if a Republican won, Anderson did.

If the material conditional theory were true, (ii) should be infelicitous (just as (iii) is), but it's not.

(iii) #I don't know whether a Republican won the election, but of course, since Reagan won, I know a Republican won.

But if *if p, q* is stronger than material implication, its falsity does not entail that *p* is true and *q* false, and hence no such implausible result is predicted.

1 Gibbard's proof

Gibbard's proof aims to establish Gibbard's conclusion by proving that the only two-place operator '$\rightarrow$' from pairs of propositions to propositions that satisfies (A1–A3) is the material conditional '$\supset$'[3]:

(A) 1. $\models p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$
2. $p \rightarrow q \models p \supset q$
3. Either $(p \not\models q)$ or $\models p \rightarrow q$

(A1–A3) are plausible principles governing indicative conditionals. Take (A1), which is sometimes called the import/export principle. It seems obvious that:

(3) a. If a Republican won the election, then if Reagan didn't win, Anderson won.
b. If a Republican won the election and Reagan didn't win, Anderson won.

are true in all the same situations, and hence equivalent.[4] And (A2) says that an indicative conditional can't be true if its antecedent is true and conclusion false, which seems correct, insofar as indicatives validate modus ponens, which they certainly seem to do. Finally, (A3) says that if its antecedent entails its consequent, the indicative must be true, which also seems right. Thus, indicative conditionals seem to be governed by (A1–A3) and from them together it follows that '$\rightarrow$' is '$\supset$', as I'll show now.[5]

Suppose $\neg p \vee q$. By disjunction elimination, we can prove $p \rightarrow q$. First disjunct: suppose that $\neg p$ is true. Since $\neg p \wedge p \models q$, by (A3), $\models (\neg p \wedge p) \rightarrow q$, and thus $(\neg p \wedge p) \rightarrow q$ is true. But by (A1), $\neg p \rightarrow (p \rightarrow q)$ is also true. Finally, since $\neg p$ is true, if $p \rightarrow q$ were false, $\neg p \rightarrow (p \rightarrow q)$ would be true and yet have a true antecedent and false consequent, which is ruled out by (A2). So, it must be that $p \rightarrow q$ is also true.

Second disjunct: suppose q is true. Since $(q \wedge p) \models q$, by (A3), $\models (q \wedge p) \rightarrow q$, and hence $(q \wedge p) \rightarrow q$ is true. Then, by (A1), $q \rightarrow (p \rightarrow q)$ is true. Since q is also true, it can't be that $p \rightarrow q$ is false, since then $q \rightarrow (p \rightarrow q)$ would be true and yet have a true antecedent and false consequent, which is ruled out by (A2). So, $(p \rightarrow q)$ must also be true. Hence, $\neg p \vee q \models p \rightarrow q$. The right-to-left direction follows directly from (A2). So indicative conditionals, which seem to obey (A1–A3), are either material conditionals or do not express propositions at all.

2 Kratzer's escape

Kratzer notices a missing lemma in the move from the conclusion of Gibbard's proof (that there is no two-place operator satisfying (A1)–(A3) except the material

[3] The material conditional $p \supset q$ is true iff p is false or q is true.

[4] Variably strict conditionals have the resources to deny this equivalence [see for instance (Stalnaker 1975)], though it's not clear they should.

[5] What follows is a crib of the version of Gibbard's proof in Gillies (2009).

conditional) to Gibbard's conclusion (that indicative conditionals are either equivalent to material conditionals or non-propositional)—this move relies on the assumption that indicative conditionals contain two-place operators in their logical forms. However, Kratzer offers an independently motivated theory of conditionals that denies this very assumption—according to her semantics, *if* does not denote a two-place operator but is rather merely a device for restricting the domains of nearby quantifiers [see Kratzer (1986, 2012)].[6] Hence, a conditional like:

(4) If John drew a one-eyed King, he must have drawn a red card.

doesn't have a logical form in which there is a conditional operator '$\rightarrow$' scoping above or below the epistemic modal *must*:

(5) a. Must: (John drew a one-eyed King $\rightarrow$ he drew a red card)
b. (John drew a one-eyed King $\rightarrow$ Must: he drew a red card)

By way of brief motivation for this kind of theory, suppose that our conditional operator is '$\supset$', as seems to be required by Gibbard's proof (if these sentences involve proposition-forming operators at all). Then (5-a) is true if it must have been the case that John didn't draw a one-eyed King and (5-b) is true if John didn't draw a one-eyed King. But neither condition is sufficient for the truth of (4), which instead requires knowing an additional fact (that there is only one one-eyed King—the King of Diamonds). Rather, (4) seems true iff

(6) It must be the case, given that John drew a one-eyed King, that he drew a red card.

Slightly more formally, this is true iff every epistemically possible world in which John drew a one-eyed King is a world at which he drew a red card. Kratzer's theory predicts this straightforwardly by having *if* restrict the domain of *must* so that it quantifies only over those worlds in which John drew a one-eyed King:

(7) [Must : John drew a one-eyed King] [John drew a red card]

(7) is true iff every epistemically possible world at which John drew a one-eyed King is a world at which he drew a red card; that is, iff (6) is true. In cases where there is no overt quantifier to restrict, Kratzer holds that *if* restricts a covert (phonologically null) quantifier, which seems, in many cases, to be an epistemic modal.[7] Thus,

[6] The inspiration for this view comes from a proposal in Lewis (1975) in which he discusses the interaction between if-clauses and adverbial quantifiers.

[7] Some conditionals seem to involve a covert habitual (or generic) quantifier over events or situations, such as:

(i) If John is in town, he visits Rudy's bar.

The indicative conditionals that are the targets of Gibbard's proof are ones like (8), and not (i) so I'll set these aside for now.

(8) If John drew a one-eyed King, he drew a red card.

has the same logical form as (4)—that is, (7).

By denying that *if* denotes a conditional operator, Kratzer's theory offers a way to accept Gibbard's proof while rejecting Gibbard's conclusion—the move is to deny that (A1)–(A3) hold of indicative conditionals. She writes, "one assumption slipped into [Gibbard's proof] that is no longer obvious: Gibbard assumes that *if . . . then* in English corresponds to a two-place propositional operator. We saw that the logical forms for natural language conditionals have a very different structure: there simply is no two-place conditional connective. *If*-clauses restrict operators" (Kratzer 2012) p. 105). However, as pointed out earlier, indicative conditionals seem to be governed by (A1)–(A3), so it's not enough to simply deny this—we must also say *why* these principles *seem* to hold of all indicative conditionals. Recognizing this, Kratzer goes on to show that her theory validates the import/export principle for indicative conditionals, a principle which we may state in schematic English as:

(9) a. *if p, then if q then r* $\equiv$
b. *if p and q*, *then r*

Kratzer's theory validates this principle by holding that both conditionals involve only a single (covert) epistemic modal that gets restricted, successively by *if p* and then *if q* in (9-a), and by the conjunctive *if p and q* in (9-b). But since domain restriction is modeled by set intersection, these amount to the same thing. Suppose that the covert modal in (9-a) and (9-b) is a universal quantifier over some set of worlds X and let P, Q, R, and $(P \cap Q)$ be the propositions denoted by *p, q, r* and *p and q* respectively. For simplicity, assume that propositions are sets of worlds and that a proposition is true at a world iff that world is a member of that proposition. Then the truth conditions of (9-a) and (9-b) amount to:

(10) a. $\forall w \in ((X \cap P) \cap Q) : w \in R$
b. $\forall w \in (X \cap (P \cap Q)) : w \in R$

But (10-a) and (10-b) are obviously equivalent, so Kratzer's theory predicts that (9-a) and (9-b) are equivalent (for all *p, q, r*), and thus validates a version of the import/export principle for indicative conditionals.

Now what about (A2) and (A3), which also seem to govern indicative conditionals? Kratzer doesn't discuss either in much detail, except to say that both are "generally accepted" (Kratzer 2012, p. 88). However, if her theory were to validate the indicative analogs of these principles, it would be in trouble, for we can construct a version of Gibbard's proof that only relies on the indicative analogs of (A1)–(A3). That is, although Gibbard's proof relies on the assumption that *if* denotes a two-place operator from pairs of propositions to propositions, this aspect of the proof can be generalized away—we can construct an analogous proof that does not rely on this assumption, and is stated instead directly in terms of the semantic values of the relevant sentence schemas.

Let $[\![p]\!]^c$ be the semantic value of the sentence p at context c. We'll assume that the semantic value of a sentence is the proposition it expresses (at that context), and also that propositions are sets of possible worlds—hence $[\![p]\!]^c$ is the set of worlds at which p is true (as uttered in c). We'll use the standard possible worlds semantics for truth, entailment and equivalence (where W is the set of all worlds and $\wp(W)$ is the set of all subsets of worlds):

(11) For all $P, Q \in \wp(W)$:

a. P is true at some $w \in W$ iff $w \in P$
b. P entails Q iff $P \subseteq Q$
c. P is equivalent to Q iff $P = Q$

We reformulate the three principles involved in Gibbard's proof without the conditional operator '$\rightarrow$' as follows:[8]

(B) For all contexts c:

1. $[\![\textit{if } p, \textit{ then if } q \textit{ then } r]\!]^c = [\![\textit{if } p \textit{ and } q, \textit{ then } r]\!]^c$
2. $[\![\textit{if } p, q]\!]^c \subseteq [\![p \supset q]\!]^c$
3. Either $([\![p^c \not\subseteq [\![q]\!]^c)$ *or* $(\wp(W) \subseteq [\![\textit{if } p, q]\!]^c)$

Let $[\![p]\!]^{c,w} = 1$ iff $w \in [\![p]\!]^c$ iff $[\![p]\!]^c$ is true at w. Here's an analogous proof that $[\![\textit{not-p} \vee q]\!]^c = [\![\textit{if } p, q]\!]^c$, which uses (B1)–(B3)—call this Gibbard*'s proof.

Suppose $[\![\textit{not-p} \vee q]\!]^{c,w} = 1$, for some arbitrary c and w. By disjunction elimination, we can prove $[\![\textit{if } p, q]\!]^{c,w} = 1$, and hence that $[\![\textit{not-p} \vee q]\!]^c \subseteq [\![\textit{if } p, q]\!]^c$, for any context c. Then by (B2) we will have proved that $[\![\textit{not-p} \vee q]\!]^c = [\![\textit{if } p, q]\!]^c$. First disjunct: suppose that $[\![\textit{not-p}]\!]^{c,w} = 1$. Since $[\![\textit{not-p and } p]\!]^c \subseteq [\![q]\!]^c$, by (B3) $\wp(W) \subseteq [\![\textit{if not-p and } p, \textit{ then } q]\!]^c$. Hence $[\![\textit{if not-p and } p, \textit{ then } q]\!]^{c,w} = 1$. By (B1), $[\![\textit{if not-p}, \textit{ then if } p, \textit{ then } q]\!]^{c,w} = 1$. Finally, since $[\![\textit{not-p}]\!]^{c,w} = 1$, if $[\![\textit{if } p, q]\!]^{c,w} = 0$, *if not-p, then if p, then q* would have a true antecedent and false consequent, which is ruled out by (B2). So it must be that $[\![\textit{if } p, q]\!]^c = 1$.

Second disjunct: suppose $[\![q]\!]^{c,w} = 1$. Since $[\![q \textit{ and } p]\!]^c \subseteq [\![q]\!]^c$, by (B3), $\wp(W) \subseteq [\![\textit{if } q \textit{ and } p, \textit{ then } q]\!]^c$. Hence, $[\![\textit{if } q \textit{ and } p, \textit{ then } q]\!]^{c,w} = 1$. Then, by (B1), $[\![\textit{if } q, \textit{ then if } p \textit{ then } q]\!]^{c,w} = 1$. Since $[\![q]\!]^{c,w} = 1$, if $[\![\textit{if } p, q]\!]^{c,w} = 0$ then *if q, then if p, then q* would have a true antecedent and false consequent, which is ruled out by (B2). So, it must be that $[\![\textit{if } p, q]\!]^{c,w} = 1$. Hence, it follows from either disjunct that $[\![\textit{if } p, q]\!]^{c,w} = 1$. Thus, $[\![\textit{not-p} \vee q]\!]^c \subseteq [\![\textit{if } p, q]\!]^c$. And the other direction is trivial [from (B2)], so we conclude that $[\![\textit{not-p} \vee q]\!]^c = [\![\textit{if } p, q]\!]^c$, for any context c.

Gibbard's conclusion follows from the conclusion of Gibbard*'s proof. Therefore, we cannot accept Gibbard*'s proof and reject Gibbard's conclusion—so, since Gibbard*'s proof is valid, the only way to reject Gibbard's conclusion is to reject one of (B1)–(B3). Thus, Gibbard*'s proof is a problem for any propositional account of indicative conditionals in which they are stronger than material implication, not just for those accounts in which *if* denotes a two-place conditional operator. We have already seen that Kratzer's theory validates (B1), so unless it

[8] In the presentation of the original Gibbard's proof, I assumed that '$\supset$' and '$\vee$' were propositional operators. In what follows below, I assume they are instead sentential operators.

invalidates one of (B2) or (B3), it does not avoid Gibbard's conclusion. In the next section, I'll show that it invalidates (B2) and argue that this is not implausible.

3 The way out

According to Kratzer's theory, indicative conditionals involve restricted covert epistemic necessity modals, where these are understood as quantifiers over a context-dependent set of possible worlds representing the possibilities compatible with what is known by some individual or group.[9] Let E be a function that takes a world w and a context c and returns a set of worlds $E(c, w)$—the c-relevant epistemically accessible worlds from that world.[10] On Kratzer's semantics, indicative if-clauses restrict the domain of this necessity modal:

(TC) $[\![\textit{if } p, q]\!]^{c,w} = 1 \text{ iff } \forall w' \in E(c,w) \cap [\![p]\!]^c : [\![q]\!]^{c,w'} = 1$

The right hand side reads: all epistemically possible p-worlds are q-worlds. Now, recall the trick we used to collapse stacked if-clauses into a single conjoined if-clause from last section. We assume that *if p, then if q, then r* and *if p and q, then r* both have a single covert modal which gets restricted by the respective if-clauses (successively in the former and all at once in the latter). Then, given the equivalence between (10-a) and (10-b), the semantics predicts the following truth conditions:

(12) $[\![\textit{if } p, \textit{ then if } q, \textit{ then } r]\!]^{c,w} = [\![\textit{if } p \textit{ and } q, \textit{ then } r]\!]^{c,w} = 1 \text{ iff}$
$\forall w' \in (E(c,w) \cap [\![p]\!]^c \cap [\![q]\!]^c) : [\![r]\!]^{c,w'} = 1$

Return to the first horn of the disjunction elimination in Gibbard*'s proof, where we've assumed $[\![\textit{not-p}]\!]^{c,w} = 1$. From this, we proved that $[\![\textit{if not-p and } p, \textit{ then } q]\!]^{c,w} = 1$ and hence by (B1) that $[\![\textit{if not-p then if } p, \textit{ then } q]\!]^{c,w} = 1$. Since $[\![\textit{not-p}]\!]^{c,w} = 1$ (by assumption) and since, by (B2), no true conditional can have a true if-clause and false consequent, we concluded that $[\![\textit{if } p, q]\!]^{c,w} = 1$. Now we can see what went wrong in this line of reasoning, according to Kratzer's semantics. For $[\![\textit{if not-p and } p, \textit{then } q]\!]^{c,w} = [\![\textit{if not-p then if } p, \textit{then } q]\!]^{c,w} = 1$, but only because:

(13) $\forall w' \in (E(c,w) \cap [\![\textit{not-p}]\!]^c \cap [\![p]\!]^{c,w}) : [\![q]\!]^{c,w'} = 1$

These truth conditions are satisfied vacuously, so $[\![\textit{if not-p and } p, \textit{then } q]\!]^{c,w} = [\![\textit{if not-p}, \textit{if } p, \textit{then } q]\!]^{c,w} = 1$, vacuously. But notice that even though $[\![\textit{not-p}]\!]^{c,w} = 1$, it may still be the case that $[\![\textit{if } p, q]\!]^{c,w} = 0$. This is because it has

[9] Stalnaker (1975); Warmbrod (1983); Weatherson (2001, 2009); Stephenson (2007); Gillies (2004, 2009, 2010) also endorse the thesis that indicative conditionals are epistemic. Strictly speaking, Kratzer rejects identifying the domain of epistemic modals with those worlds compatible with the knowledge of some group but instead identifies it with the worlds compatible with some body of information (see Kratzer (2009)). This information may be encoded in the contents of a closed filing cabinet, for instance. This detail won't matter for our purposes.

[10] We're ignoring a host of complications here that don't concern us. The only relevant feature of epistemic modals needed here is that they involve universal quantification over a set of worlds.

the truth conditions stated in (TC). Here's an explicit counterexample: let $E(c,w) = \{w, w_1, w_2\}$, $[\![not\text{-}p]\!]^c = \{w\}$, $[\![p]\!]^c = \{w_1, w_2\}$ and $[\![q]\!]^c = \{w_1\}$. Then:

(14) a. $[\![\textit{if not-p, then if p, then q}]\!]^{c,w} = 1$
-Since $\forall w' \in (E(c,w) \cap [\![not\text{-}p]\!]^{c,w} \cap [\![p]\!]^c) : [\![q]\!]^{c,w'} = 1$, vacuously
-Since $\{w, w_1, w_2\} \cap \{w\} \cap \{w_1, w_2\} = \varnothing$
b. $[\![not\text{-}p]\!]^{c,w} = 1$
c. $[\![\textit{if p, q}]\!]^{c,w} = 0$
-Since $\exists w' \in E(c,w) \cap [\![p]\!]^c : [\![q]\!]^{c,w'} = 0$; the world is w_2

Thus, we see that Kratzer's semantics avoids Gibbard's conclusion by denying (B2). Furthermore, this is not surprising, since when the sentence *if p, q* is embedded in the consequent of an indicative conditional, it doesn't denote a proposition on Kratzer's theory, yet when it appears embedded under a material conditional, it does. Thus, when *if q, r* is substituted for *q* in (B2) (reprinted below), the resulting instance of (B2) will be false.

(B2) $[\![\textit{if p, q}]\!]^c \subseteq [\![p \supset q]\!]^c$
$[\![\textit{if p, then if q, then r}]\!]^c \not\subseteq [\![p \supset \textit{if q, then r}]\!]^c$

Notice that we block the derivation in the second disjunct of Gibbard's proof this way as well. Assume $[\![q]\!]^{c,w} = 1$. We can then prove that $[\![\textit{if q and p, then q}]\!]^{c,w} = [\![\textit{if q, then if p, then q}]\!]^{c,w} = 1$. This is so on Kratzer's semantics too. But it doesn't follow from this and (B2) that since $[\![q]\!]^{c,w} = 1$, $[\![\textit{if p, q}]\!]^{c,w} = 1$. Here is a counterexample: let $E(c,w) = \{w, w_1, w_2\}$, $[\![q]\!]^c = \{w\}$, $[\![p]\!]^c = \{w, w_1\}$. Then:

(15) a. $[\![\textit{if q, then if p, then q}]\!]^{c,w} = 1$
-Since $\forall w' \in (E(c,w) \cap [\![q]\!]^c \cap [\![p]\!]^c) : [\![q]\!]^{c,w'} = 1$
b. $[\![q]\!]^{c,w} = 1$
c. $[\![\textit{if p, q}]\!]^{c,w} = 0$
-Since $\exists w' \in E(c,w) \cap [\![p]\!]^c : [\![q]\!]^{c,w'} = 0$; the world is w_1

Thus, Kratzer's theory predicts counterexamples to (B2) and hence avoids Gibbard's conclusion.

4 Lessons

Notice that any theory in which indicative conditionals are universal quantifiers over possible worlds will supply counterexamples to (B2). Let *X* be the set of worlds being quantified over. Then whenever $X \cap [\![p]\!]^c = \varnothing$, the following truth conditions are satisfied:

(16) $[\![\textit{if p, q}]\!]^{c,w} = 1 \text{ iff } \forall w' \in (X \cap [\![p]\!]^c) : [\![q]\!]^{c,w'} = 1$

But whenever $X \cap [\![p]\!]^c = \varnothing$ and $[\![p]\!]^c \neq \varnothing$, we can cook up counterexamples to (B2). That is because there will be *p*-worlds around, and if any of them are also ¬*q*-

worlds, then they will be worlds at which *if p, q* is true (because it is true vacuously) and in which $p \supset q$ is false.

By itself, this isn't enough to avoid Gibbard's conclusion. For notice that in the first disjunct of Gibbard*'s proof, we appealed to a conditional (*if p and not-p, then q*) which was vacuously true because there are no $(p \wedge \neg p)$-worlds. But in this case we won't be able to find a world at which $\neg p$ is true and $p \supset q$ false—since the former entails the latter. Hence, if there are no worlds in which *if p, q* is true and $p \supset q$ is false, *if p, q* must be true in any $\neg p$ world. So it's not enough to avoid Gibbard's conclusion that your semantics predicts that (B2) may be false in cases where $X \cap [\![p]\!]^c = \emptyset$ and $[\![p]\!]^c \neq \emptyset$, since Gibbard*'s proof doesn't rely on such cases.

The way Kratzer's theory falsifies (B2) is more radical—it has false instances due to equivocations (when q is itself an indicative conditional it does not express a proposition in *if p, q* but it does in $p \supset q$), and these equivocations are exactly the ones Gibbard*'s proof exploits. In the Appendix, I show why falsifying (B2) in this way allows Kratzer's theory to escape the conclusion of the revamped version of Gibbard's original presentation of the proof as well.

But recall that (B2) seemed intuitively correct—for instance, it is what ensures that indicative conditionals validate the semantic counterpart of modus ponens. So, isn't it a cost of the semantics that it predicts that (B2) has false instances? Kratzer's semantics validates (B2) for values of q that are not themselves indicative conditionals, and hence predicts that for non-conditional values of q, an indicative conditional *if p, q* can't be true if p is true and q false. However, it also predicts that *if p, then if q, then r* and p can both be true and *if q, then r* be false; thus, it predicts that the semantic counterpart of modus ponens is classically invalid for indicative conditionals with indicative conditionals embedded in their consequents [cf. McGee(1985)'s counterexamples].[11]

But perhaps even this limited denial of modus ponens is too steep a cost to pay for propositionalism about indicative conditionals. Perhaps, but it's not obviously a prohibitive cost either. For one, notice that Anthony Gillies' theory of conditionals also avoids Gibbard's conclusion by rejecting (B2). According to Gillies' theory, *if* denotes a two-place conditional operator that shifts both the index (world) and context relative to which the consequent is evaluated. Shifting the context relative to which the consequent is evaluated amounts to potentially shifting the semantic value of any context-dependent expressions in the consequent—importantly, for our purposes, the value of the domain of epistemically accessible worlds indicative conditionals quantify over. The "shifted" domain is defined to be identical to the original domain restricted to worlds in which the *if*-clause is true, thus mimicking the *if*-clause's restricting feature in Kratzer's theory. Formally:

(17) $[\![\textit{if } p, q]\!]^{c,w} = 1 \text{ iff } \forall w' \in E(c+p, w) : [\![q]\!]^{c+p,w'} = 1$

[11] This is a consequence of the fact that it validates (B1) and assigns indicative conditionals truth conditions stronger than material implication. Here's a simple counterexample to modus ponens with embedded indicatives:

- $E(c,w) = \{w, w_1, w_2, w_3\}, [\![p]\!]^c = \{w, w_2\}, [\![q]\!]^c = \{w_2, w_3\}, [\![r]\!]^c = \{w_2\}$
- Then $[\![\textit{if } p, \textit{ then if } q, \textit{ then } r]\!]^{c,w} = 1$ and $[\![p]\!]^{c,w} = 1$.
- But $[\![\textit{if } q, \textit{ then } r]\!]^{c,w} = 0$; since $w_3 \in E(c,w) \cap [\![q]\!]^c$ and $w_3 \notin [\![r]\!]^c$.

where $E(c + p, w)$ is defined as follows:

(18) For any $c, w, p : E(c + p, w) = E(c, w) \cap [\![p]\!]^c$

Given these definitions, and the assumption that E is well-behaved:

(19) E is well-behaved iff for all c, w, w' : if $w' \in E(c, w)$ then $E(c, w) = E(c, w')$.

Gillies' theory predicts the equivalence of *if p, then if q, then r* and *if p and q, then r* [see Gillies (2009, p. 331)], but thereby also predicts that *if p, then if q, then r* and *p* can both be true while *if q, then r* false. Here's a counterexample that mirrors the one from footnote 11:

- $E(c, w) = \{w, w_1, w_2, w_3\}, [\![p]\!]^c = \{w, w_2\}, [\![q]\!]^c = \{w_2, w_3\}, [\![r]\!]^c = \{w_2\}$
- Then $[\![p]\!]^{c,w} = 1$, and $[\![\textit{if } p, \textit{ then if } q, \textit{ then } r]\!]^{c,w} = 1$ (see derivation here)

  $[\![\textit{if } p, \textit{ then if } q, \textit{ then } r]\!]^{c,w} = 1$ iff

  $\forall w' \in E(c + p, w) : [\![\textit{if } q, \textit{ then } r]\!]^{c+p,w'} = 1$ iff

  $\forall w' \in E(c + p, w) : \forall w'' \in E(c + p + q, w') : [\![r]\!]^{c+p+q,w''} = 1$ iff

  $\forall w' \in (E(c, w) \cap [\![p]\!]^c) : \forall w'' \in (E(c, w') \cap [\![p]\!]^c \cap [\![q]\!]^c) : [\![r]\!]^{c+p+q,w''} = 1$ iff
  Definition of $E(c + p, w)$

  $\forall w' \in (E(c, w) \cap [\![p]\!]^c) : \forall w'' \in (E(c, w) \cap [\![p]\!]^c \cap [\![q]\!]^c) : [\![r]\!]^{c+p+q,w''} = 1$
  Well-behaved

  $\forall w' \in (E(c, w) \cap [\![p]\!]^c \cap [\![q]\!]^c) : [\![r]\!]^{c+p+q,w'} = 1$
  Vacuous quantification

- But $[\![\textit{if } q, \textit{ then } r]\!]^{c,w} = 0$; since $w_3 \in E(c, w) \cap [\![q]\!]^c$ and $w_3 \notin [\![r]\!]^c$.

Thus, both Kratzer and Gillies' theories avoid Gibbard's conclusion by rejecting (B2). However, although both thereby invalidate modus ponens for indicatives with embedded conditionals in their consequents, both theories are consistent with the following fallback position regarding modus ponens: when running through an argument, one asserts or supposes its premises, so if asserting or supposing *p* (or the proposition it expresses) restricts the domain of indicative conditionals $E(c, w)$ to *p*-worlds, it will be impossible to assert (or suppose) both *if p, then if q, then r* and *p* without thereby ensuring that *if q then r* expresses something true (as evaluated under those suppositions/assertions—i.e., with the domain of *if q then r* restricted to *p*-worlds by the supposition/assertion of *p*). Thus, both Kratzer's and Gillies' semantics are compatible with modus ponens involving embedded indicatives being "dynamically" valid [cf. Stalnaker 1975's "reasonable inference" and Gillies (2004, pp. 592–595)].

The foregoing alleviates some of the cost to predicting counterexamples to (B2). First, Kratzer's theory does not predict counterexamples to (B2) involving simple indicatives but rather only those embedding indicative conditionals in their consequents, just as McGee's counterexamples to modus ponens independently confirm. Second, Gillies' theory, which also avoids Gibbard's conclusion, faces the same cost. Finally, it's compatible with both theories that, although modus ponens is not classically valid, it is dynamically valid. Thus, both Kratzer's and Gillies' theories remain viable propositionalist theories of indicative conditionals that avoid Gibbard's conclusion.

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Appendix

For readability, I reprint (B1)–(B3) here:

(B) For all contexts c:

1. $[\![\textit{if } p, \textit{ then if } q \textit{ then } r]\!]^c = [\![\textit{if } p \textit{ and } q, \textit{ then } r]\!]^c$
2. $[\![\textit{if } p, q]\!]^c \subseteq [\![p \supset q]\!]^c$
3. Either $([\![p]\!]^c \not\subseteq [\![q]\!]^c)$ or $(\wp(W) \subseteq [\![\textit{if } p, q]\!]^c)$

Here is a reconstruction of Gibbard's original presentation of his proof that $[\![p \supset q]\!]^c \subseteq [\![\textit{if } p, q]\!]^c$, for any context c, using (B1)–(B3). Consider the following conditional:

(G) *if* $(p \supset q)$, *then if p, q*

In what follows, we'll make use of an arbitrary context c. By (B1), $[\![\textit{if } (p \supset q), \textit{ then if } p, q]\!]^c = [\![\textit{if } (p \supset q) \textit{ and } p, \textit{ then } r]\!]^c$. Since $[\![(p \supset q) \textit{ and } p]\!]^c \subseteq [\![q]\!]^c$, it follows from (B3) that $\wp(W) \subseteq [\![\textit{if } (p \supset q) \textit{ and } p, \textit{ then } r]\!]^c$ and hence that $\wp(W) \subseteq [\![\textit{if } (p \supset q), \textit{ then if } p, q]\!]^c$. That is, $[\![(\text{G})]\!]^{c,w} = 1$ for any w. Now, by substituting appropriately into (B2), we get that $[\![\textit{if } (p \supset q), \textit{then if } p, q]\!]^c \subseteq [\![(p \supset q) \supset \textit{if } p, q]\!]^c$. But since $[\![(\text{G})]\!]^c$ is necessarily true, and since $[\![(\text{G})]\!]^c$ entails $(p \supset q) \supset \textit{if } p, q$, then the latter is also necessarily true. But that means that there are no worlds in which $(p \supset q)$ is true and *if p, q* false—hence, $[\![p \supset q]\!]^c \subseteq [\![\textit{if } p, q]\!]^c$.

In this version of the proof, the key step is similar to Gillies' version discussed earlier—it is the step where we substitute into (B2). Kratzer's semantics validates the reasoning up to the step where $\wp(W) \subseteq [\![\textit{if } (p \supset q), \textit{ then if } p, q]\!]^c$. But the semantics allows for models in which there is a world w such that $[\![\textit{if} (p \supset q), \textit{ then if } p, q]\!]^{c,w} = 1$ and $[\![(p \supset q) \supset \textit{if } p, q]\!]^{c,w} = 0$. Here is one: $E(c,w) = \{w, w_1, w_2\}$, $[\![p]\!]^c = \{w_1\}$, $[\![q]\!]^c = \{w\}$, and $[\![p \supset q]\!]^c = \{w, w_2\}$. Then:

(20) a. $[\![\textit{if} (p \supset q), \textit{then if } p, q]\!]^{c,w} = 1$
-Since $\forall w' \in (E(c,w) \cap [\![p \supset q]\!]^c \cap [\![p]\!]^c) : [\![q]\!]^{c,w'} = 1$, vacuously
-Since $\{w, w_1, w_2\} \cap \{w, w_2\} \cap \{w_1\} = \varnothing$

b. $[\![(p \supset q) \supset \textit{if } p, q]\!]^{c,w} = 0$
-Since $[\![p \supset q]\!]^{c,w} = 1$ and $[\![\textit{if } p, q]\!]^{c,w} = 0$
-The latter holds because $\exists w' \in E(c,w) \cap [\![p]\!]^c : [\![q]\!]^{c,w'} = 0$; the world is w_1

Hence, by predicting that the required substitution instance of (B2) is in fact false, Kratzer's semantics avoids the conclusion of Gibbard's original presentation of the proof as well.

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