Logical remainders and indicative conditionals

Justin Khoo
justin.khoo@yale.edu

July 19, 2013

In his remarkable new book, *Aboutness* (Yablo 2013), Stephen Yablo makes a compelling case that sometimes the *felt* truth value or content of a sentence is not its semantic content but rather its semantic content *minus* a related presupposition (the result of logically subtracting a relevant presupposition from its semantic content). Yablo uses this theory to provide a unified account of non-catastrophic presupposition failures, cheap ontological arguments, and cases of unexpected assertive content.[1] This proposed unification is surprising, illuminating, and really, quite exciting. But what is the notion of logical subtraction underlying the theory—what is it to *logically subtract* some content from another? And do we have an expression of ordinary language that picks out logical remainders (what's left after an operation of logical subtraction has been carried out)? In this paper, I propose that indicative conditionals might be our way of expressing logical remainders. If correct, we'll have some new resources of thinking about both conditionals and remainders. And even if indicative conditionals are not equivalent to their corresponding logical remainders, I'll argue that they are pretty closely related—close enough to warrant thinking about what one can learn from one by way of the other.

My strategy will proceed both at the intuitive level and by way of a technical comparison of my favored semantics for indicatives and Yablo's truthmaker semantics for logical remainders. Here's the plan for what follows. In §1, I review Yablo's informal discussion of logical subtraction and its role in explaining non-catastrophic presupposition failures, cheap ontological arguments, and cases of unexpected assertive content. In §2, I argue that indicative conditionals are apt to play a very similar role with respect to such phenomena. In §3, I discuss the challenge of providing truth conditions to logical remainders, and argue

[1] Yablo also puts logical subtraction to work in defining concepts by overshooting and backtracking and delivering the assertoric effect of epistemic possibility claims.

that an analogous challenge faces indicative conditionals. In §4, I propose a semantics for indicative conditionals that I think can resolve the challenge. Finally, in §5 I compare the resulting semantics for indicatives with the Yablo's truthmaker semantics for logical remainders, and argue that, although they are not equivalent, they are very closely related.

1 What is logical subtraction?

Let's begin with some intuitions.

Non-catastrophic presupposition failure. Strawson 1954 pointed out some cases in which uttering a sentence whose presupposition fails in that context doesn't always lead to catastrophe (non-evaluability of the asserted content). Here's an example from Yablo 2006, gesturing to an empty chair:

(1) The King of France is sitting in that chair.

Unlike its Russellian counterpart (2), (1) seems intuitively false—that chair is empty, which we can all see and are mutually presupposing.

(2) The King of France is bald.

The puzzle is: why are some presupposition failures (like that of (1)) non-catastrophic and others (like (2) perhaps) catastrophic? An intuitive idea is that (1) is false for reasons not having to do with whether there is a King of France—that's on the right track, but it remains to be seen how to spin this idea into a theory.

Cheap ontological arguments. There's something cheap about ontological arguments like this one:

(3) a. The number of planets is greater than zero.
  b. If the number of planets is greater than zero, then there are numbers.
  c. So, there are numbers.

Such arguments proceed from the widely accepted truth of (3-a) and (3-b) via the validity of the argument form of (3). But this seems too fast! The challenge is to diagnose what is wrong with such cheap arguments. An intuitive idea is that a switch occurs when moving from (3-a) to (3-b)—we are willing to go along with (3-a), not worrying about whether there are numbers, until we realize (3-b), and our worries about whether there are numbers start to kick in. Again, this is on the right track, but it remains to be seen how to spin this idea into a theory.

Unexpected content. Before Donnellan 1966, it would have been natural to think that the expression "Smith's murderer" can only be used to refer to the person who murdered Smith. However, Donnellan showed that as long as you're in a context with the right presuppositions in place, you can refer to just about anyone by that expression. The question then is: why would that be so? How could an utterance of

(4) Smith's murderer is insane.

come to communicate that Joe is insane (where Joe is the person presupposed among ones interlocutors to be Smith's murderer), even in a world in which Smith is still alive (though this fact must remain unknown)? An intuitive idea is that we somehow compute the unexpected asserted content by way of what would make an appropriate addition to our mutual information, given what we're all taking for granted—as before, not a theory, but an intriguing starting point.

Enter: logical subtraction. Yablo's hypothesis is that logical subtraction is the key to spinning the sketchy ideas about the above puzzle cases into an explanatory theory. But what is logical subtraction? Let A and B be the truth-conditional contents (propositions, sets of possible worlds) of the sentences A and B, respectively, and follow this convention throughout. Let '$-$' be the logical subtraction operation on propositions such that $A - B$ is A, with B logically subtracted from it. Alternatively, we may think of this operation in terms of what it yields—logical remainders; so, $A - B$ is the logical remainder of A once B is subtracted from it. We stipulate that logical subtraction is, at the very least, an undoing of logical addition, and our paradigm case of logical addition is conjunction: $A + B = A \wedge B$. Hence, we demand at least if you add B back to $A - B$ you get A—hence, $(A - B) \wedge B \models A$. Unfortunately, not all contents are built up like conjunctions, so computing the truth conditions for logical remainders is messier than doing the same for logical sums. More on this below.

Here's another way to get an intuitive grip on logical subtraction. Consider the set of worlds that comprise logical space, and thus propositions as occupying regions of logical space (the regions occupied by all and only those worlds in logical space at which they are true). On this understanding, propositions impose constraints on logical space—to believe that A is true is to think oneself among the A-worlds. Suppose for now that A entails B—hence, the A-region is entirely contained within the B-region. What happens when we logically subtract B from A? Intuitively, we extend A *beyond* the B-region—into the $\neg B$-region, that is—as in Figure 1:

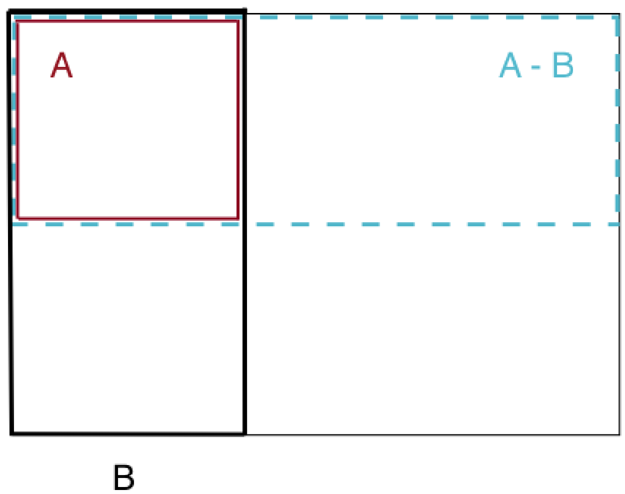

Figure 1

Just how far $A - B$ extends into the $\neg B$-region is the question we'll take up in §3. For now, let's consider an intuitive example (I'll leave a more precise discussion of Yablo's truthmaker semantics for remainders to §5):

(5) The King of France is a monarch. (M)

Let M be the propositional content of (5), and K be the propositional content of (6), which (5) presupposes:

(6) There is a King of France. (K)

What, then, is the propositional content of $M - K$? Well, let's make two simplifying assumptions. First, all presuppositions are entailments (though not vice versa!)—so M is false at worlds in which K is. Second, that every king is a monarch is necessarily true. How could we evaluate $M - K$? Yablo offers an intriguing suggestion: think about what would make the material conditionals $K \supset M / K \supset \neg M$ true at the various worlds. In particular, are there ways of making $K \supset M / K \supset \neg M$ true that are compatible with K? Since the material conditional $K \supset M$ is true if K is false or M is true, if there's a sufficient ground for the truth of $K \supset M$ that is compatible with K, it seems that this would go some way towards the truth of $M - K$. Here's how Yablo puts this idea to work in specifying (roughly) the truth conditions of $M - K$:

(7) a. $M - K$ is true at w if there are K-compatible truthmakers for $K \supset M$ at w but no K-compatible truthmakers for $K \supset \neg M$ at w.
b. $M - K$ is false at w if there are K-compatible truthmakers for $K \supset \neg M$ at w but no K-compatible truthmakers for $K \supset M$ at w.
c. $M - K$ is undefined at w otherwise.

The basic idea is to check all the K-compatible truthmakers—all the propositions that have a non-empty intersection with K—with respect to $K \supset M$ and $K \supset \neg M$. Four conditions exist: some are truthmakers for $K \supset M$ and none for $K \supset \neg M$, some are truthmakers for $K \supset \neg M$ and none for $K \supset M$, some for both, and none for either. In the first case $M - K$ is true, the second $M - K$ is false, and the third and fourth $M - K$ is undefined. Notice that, given our stipulation that necessarily all kings are monarchs, this necessary truth is a K-compatible truthmaker for $K \supset M$. Furthermore, since this truthmaker obtains at every world, it makes $K \supset M$ true at all worlds, and hence $M - K$ is necessarily true. We're well on our way to seeing how logical remainders can provide a theory of non-catastrophic failure.

Recall the contrast between (1) and (2):

(1) The King of France is sitting in that chair. (C) False

(2) The King of France is bald. (B) Unevaluable

Yablo's hypothesis is that the felt truth value of a sentence is often the truth value of its semantic content minus its presupposed content. Hence, the felt truth value of C/B—the proposition expressed by (1)/(2)—is the real truth value of $C - K/B - K$. And the reason we judge (1) false and (2) unevaluable is that $C - K$ is false while $B - K$ is undefined. Let's see why one at a time.

We're at the actual world α at which there is no King of France and at which that chair is unoccupied. Is there a K-compatible truthmaker for either $K \supset C$ or $K \supset \neg C$? Not for the former: it's only made true by the fact that $\neg K$, which is not K-compatible. But the latter has a K-compatible truthmaker: the emptiness of the chair suffices to make $K \supset \neg C$ true, and is compatible with K (since it's possible for the chair to be empty and for there to be a King of France). Hence $K \supset \neg C$ has a K-compatible truthmaker at α and $K \supset C$ doesn't—so we predict that $C - K$ is false at α.

What about $B - K$? Well, there don't seem to be any K-compatible truthmakers for either $K \supset B$ or $K \supset \neg B$—nothing in the fabric of the cosmos ensures that the King of France must be bald, for instance. Thus, we predict that $B - K$ is undefined (neither true nor false), and this is how we account for our intuition that (2) is unevaluable (or truth valueless).

There's an analogous resolution of the puzzle of cheap ontological arguments (such as (3) reprinted here):

(3) a. The number of planets is greater than zero. (Z)
b. If the number of planets is greater than zero, then there are numbers. ($Z \supset N$)
c. So, there are numbers. (N)

The challenge is to explain why we think (3-a) is true and yet don't think this commits us to (3-c) being true, despite our acceptance of (3-b) and the validity of (3)-style arguments. A possible answer to the challenge is that our intuitions about the truth value of (3-a) track, not Z (its semantic content, which entails that there are numbers, N) but rather $Z - N$. And since $N \supset Z$ has an N-compatible truthmaker—the fact that there are some planets—and $N \supset \neg Z$ has no N-compatible truthmakers, we judge $Z - N$ to be true. This strategy is akin to ones that posit ambiguity to explain cheap ontological arguments, except without positing the ambiguity—the felt truth value of (3) is that of $Z - N$, even though its actual truth value is just that of Z. It's just that we typically care about the truth value of $Z - N$ rather than Z (just as is confirmed by our judgments of non-catastrophic presupposition failures).

In his discussion of unexpected assertoric content, Yablo offers another avenue to logical remainders, by way of interpolation. I'll review the intuitive case for interpolants as remainders here. Consider the following enthymematic argument:

(8) a. All men are mortal.

b. ??

c. Socrates is mortal.

There are many things to fill in for (8-b) to make (8) a valid argument: Socrates is a mortal man, Socrates is not immortal, and so on. But one interpolates between (8-a) and (8-c) by being such that it *explains* (8-c)'s truth, together with (8-a), rather than merely ensures it: Socrates is a man. Yablo argues that what interpolates between some conclusion C and premise A in general just is $C - A$. This is intuitively plausible: remember, $(C - A) \wedge A$ just is C. So, remainders provide just the right amount of content to get from a premise A to a conclusion C—thus, intuitively, remainders are interpolants. Let's leave the discussion at the intuitive level for now and return to Donnellan's referential definite description:

(4) Smith's murderer is insane.

Yablo proposes that we can explain why the felt (or asserted) content of (4) is that Joe is insane (in a context in which Joe is presupposed to be Smith's murderer) as follows. First, we set up an argument whose major premise is the background presupposition and whose conclusion is the semantic content of (4):

(9) a. Joe is Smith's murderer. J

b. ??

c. Smith's murderer is insane. S

Yablo proposes that what interpolates between (a) and (c) here is the felt (asserted) content of (4) in this context. And, intuitively, what interpolates (a) and (c) is just that Joe is insane, which is the unexpected assertoric content of (4) in this context. Where S is the semantic content of (9-c) and J is the semantic content of (9-a), the content of (b) is $S - J$, which is plausibly just that Joe is insane. As before, the felt content (or truth value) of (9-c) is not (that of) S but $S - J$, where J is the relevant operant presupposition in the context.

Thus, we've seen how logical remainders can provide an illuminating explanation of three otherwise disparate puzzle cases. In the next section, I will begin building my case for indicative conditionals as logical remainders by showing that they seem to play the same role as logical remainders in Yablo's theory.

2 Indicative conditionals

Call the proposal we'll develop and defend the **Indicative Hypothesis**:

(10) **The Indicative Hypothesis**: The proposition expressed by the indicative conditional ⌜if B, then A⌝, *if* B, A, plays the same role as the logical remainder $A - B$ in Yablo's theory.[2]

In this section, we'll review the three cases discussed in §1 to see how well indicatives fare in the role Yablo gives to logical remainders.

Non-catastrophic presupposition failure. Recall (1) and (2):

(1) The King of France is sitting in that chair. (C) False

(2) The King of France is bald. (B) Unevaluable

Yablo's explanation why (1) seems false while (2) seems unevaluable (or truth valueless) is that the felt truth value of (1)/(2) is $C - K/B - K$. Is it plausible that the felt truth values of (1)/(2) are those of (11)/(12) respectively?

(11) If there is a King of France, then he's sitting in that chair.

(12) If there is a King of France, then he's bald.

For what it's worth, my intuition is that (11) is false, while (12) is unevaluable (hard to evaluate, at best neither true nor false). For (11), it seems natural to reason as follows: well, whether or not there's a King of France, that chair is unoccupied, so he (if there is such a he) isn't in that chair, so (11) is false. For (12), I cannot run any similar sort of argument to convince myself that it's true or false. Now, notice that, if we consider the indicative version of (5) instead—(13)—and remind ourselves of the necessity that all kings are monarchs, it becomes easier to run an analogous case that (13) is true.

(5) The King of France is a monarch.

(13) If there is a King of France, he's a monarch.

[2] It's controversial whether indicative conditionals express propositions, partly because of considerations we'll discuss in §3. See Gibbard 1981, Edgington 1995, Bennett 2003 for some of the challenges, and Rothschild 2011, Khoo 2013b for some responses.

So far, our intuitions about the corresponding indicative conditionals (if they are such) match our felt intuitions about the presupposition failing sentences (1), (2), and (5). Thus, it seems plausible that the corresponding indicative conditionals may play a similar role as logical remainders in Yablo's theory of non-catastrophic presupposition failure.

Cheap ontological arguments. Recall our cheap argument from §1:

(3) a. The number of planets is greater than zero. (Z)
b. If the number of planets is greater than zero, then there are numbers. ($\mathit{if}\,\neg Z, N$)
c. So, there are numbers. (N)

Yablo's diagnosis is that the felt truth value of (3-a) is the actual truth value of $Z - N$, which is true, and does not entail that there are numbers (that entailment of Z has been subtracted from it). By the **Indicative Hypothesis**, we predict that $\mathit{if}\,N, Z$ ought to be true, if it's to play the same role in Yablo's theory as its corresponding remainder; furthermore, insofar as Yablo's theory strikes us as plausible, it would be promising if it were plausible that the felt truth value of (3-a) goes by the actual truth value of $\mathit{if}\,N, Z$. To the former, consider (14):

(14) If there are numbers, then the number of planets is greater than zero.

This strikes me as true simply in virtue of there being some planets. Indeed, it strikes me as true whether or not there are numbers. This is because the presupposition of (3-a) is locally satisfied by the if-clause—it does not project out of the consequent into the sentence as a whole but is rather filtered by the if-clause, since the latter entails it. Indeed, the ability to filter presuppositions is a well-known feature of indicative conditionals, first pointed out (I think) by Karttunen 1973 but recognized well before that by anyone pressed about presuppositions they don't care much about:

(15) (a conversation between two priests about divine properties)
a. Priest 1: God is all-knowing, and this poses trouble for free will.
b. Agnostic (interjecting): but how do you know God exists?
c. Priest 2: that's not our concern right now. What we're concerned about is whether, if the God of the Old and New Testaments exists, God is all-knowing.

Thus not only does (14) seem true for the reasons we might intuitively and naively think (3-a) is true, it is not implausible that this is so, for indicative antecedents filter presuppositions and are thus may be used to avoid commitment to presuppositions one doesn't care much about.

Unexpected content. Recall Donnellan's example:

(4) Smith's murderer is insane.

Yablo's explanation of how (4) might come to mean something closer to that Joe is insane is that that content is what interpolates between the semantic content of (4) and a background presupposition that Joe is Smith's murderer:

(16) a. Joe is Smith's murderer. (J)
b. Joe is insane. ($S - J$)
c. Smith's murderer is insane. (S)

By the **Indicative Hypothesis**, we predict that *if* J, S should be roughly equivalent to $S - J$, that is, (16-b), in such a context. And, indeed, it strikes me as plausible that (17) is roughly equivalent to (16-b).

(17) If Joe is Smith's murderer, then Smith's murderer is insane.

Neither (16-b) nor (17) (nor $S - J$) entails that Joe is Smith's murderer. Whereas (4) relies on the background presupposition J to communicate that (16-b), (17) simply makes the relevant background presupposition J explicit. One concern with the equivalence is that (17) might be true simply because Smith's murderer is insane, whether or not Joe is Smith's murderer (this is so on its concessive reading). However, that's not what we typically aim to communicate when we utter an indicative like (17)—often, we intend to communicate something stronger, which *is* intuitively equivalent to (16-b). Part of the explanation for why the concessive reading is typically ruled out is that indicatives seem to conversationally implicate that the speaker is ignorant with respect to the conditional's consequent. As long as not all of the relevant worlds are ones in which Smith's murderer is insane, the concessive reading of (17) is ruled out. In §4, I'll show how a plausible semantics for indicatives predicts the standard reading of (17) to be equivalent to (16-b).[3]

[3] It may help to review one more case Yablo discusses, to better press the case for indicatives here. If I utter (i) in a context in which it's common ground that my cousin has not changed genders, I end up

Our review of Yablo's theory of non-catastrophic presupposition failure, cheap ontological arguments, and unexpected assertoric content reveals that ordinary indicative conditionals may be fit to play the same role as logical remainders. This is our first pass at bringing the two together. Let's now compare some proposals for the truth conditions of each.

3 An analogous challenge

Yablo discusses a standing problem (*the* problem?) for assigning truth conditions to logical remainders. $A - B$ seems to be logically weaker than $A \wedge B$ and logically stronger than $B \supset A$—in other words, $A \wedge B$ entails $A - B$ but not vice versa, and $A - B$ entails $B \supset A$ but not vice versa. Why?

(i) $A \wedge B \models A - B$. Intuitively, $A \wedge B$ contains all the information (and more!) of $A - B$, so if the former is true, the latter must be.

(ii) $A - B \not\models A \wedge B$. Simply put, given the above entailment, if this held $A - B$ would be equivalent to $A \wedge B$. But then logical subtraction would be equivalent to logical addition, which is absurd. Alternatively, $A - B$ extrapolates A beyond the B-region. But if it is to extend anywhere, it must be true at some $\neg B$-worlds. Hence, $A - B$ can't entail $A \wedge B$.

(iii) $A - B \models B \supset A$. Recall that when you add B back to $A - B$, you get A: $A - B, B \models A$. Thus, a version of modus ponens must hold for logical remainders, which means that

communicating that my cousin has grown up:

(i) My cousin is not a boy anymore.

As before, we set up an argument whose conclusion is the semantic content of (i) and premises are the background presupposition and the assertoric (felt content) of (i):

(ii)
a. My cousin is still a male human. M
b. My cousin is grown up. $\neg B - M$
c. My cousin is not a boy anymore. $\neg B$

It's not a very natural thing to say, but I think it's not unreasonable that (iii) mean roughly the same as (b).

(iii) If my cousin is still a male human, then my cousin is not a boy anymore.

$\approx$ My cousin is grown up.

they must entail their converse material conditionals.

(iv) $B \supset A \not\models A - B$. If this held, then $A - B$ would be equivalent to $B \supset A$, in which case extrapolating A beyond B would fill up the entire $\neg B$-region of logical space. But this is too weak. Intuitively, subtracting a conjunct from a conjunction should leave us with the other conjunct, $(A \wedge B) - A = B$. However, if $A - B = B \supset A$, we'd have: $(A \wedge B) - A = A \supset (A \wedge B)$. But $A \supset (A \wedge B)$ is weaker than B—the former is entailed by $\neg A$, which the latter is not.

Figure 1 may help to clarify the situation:

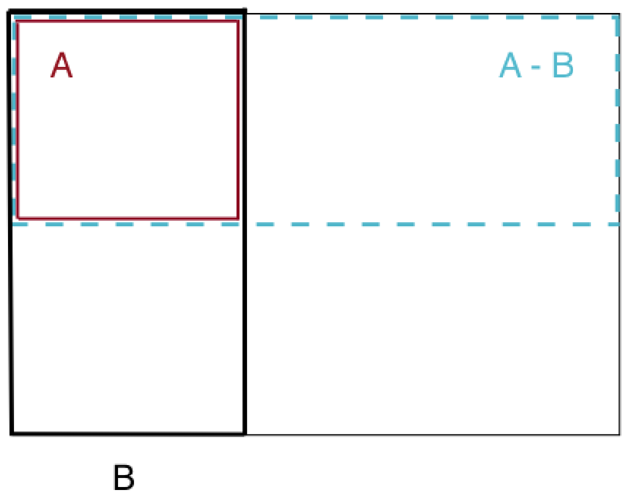

Figure 2

(We're need no longer assume that $A \models B$.) Notice that $A - B$ extends beyond the B-region somewhat (hence $A - B \not\models A \wedge B$), but doesn't fill up the entire $\neg B$-region (hence $B \supset A \not\models A - B$). We might summarize the truth table for $A - B$ as follows:

The challenge is to figure out some principled recipe for determining just how far $A - B$ takes us beyond B, or, what considerations determine the truth value for $A - B$ when B is false.

Let's turn to indicative conditionals—I hope to show now that we face an analogous challenge in formally stating their truth conditions. We'll show that indicatives arguably have each of the four properties of logical remainders above:

B	A	$A - B$
T	T	T
T	F	F
F	T	?
F	F	?

(i) $A \wedge B \models \mathit{if}\, B, A$. This is the most controversial of the four properties, itself an entailment of the principle better known in the conditionals literature as "strong centering".[4] The most compelling argument for this entailment goes by our betting intuitions. Suppose John has just rolled a fair six-sided die and then utters:

(18) If I just rolled an even number, I rolled a prime.

I bet that what John said is true, and you bet against me. It seems that if John rolled an even prime (a 2) then I win the bet, and if John rolled an even non-prime (either a 4 or 6) then you win the bet (never mind for now who wins if John rolled an odd number). If our intuitions about the bets here are a reliable guide to our intuitions about the truth conditions of indicative conditionals, we have a good reason to endorse the first property.[5]

(ii) $\mathit{if}\, B, A \not\models A \wedge B$. Given (i), if $\mathit{if}\, B, A$ did entail $A \wedge B$, then $\mathit{if}\, B, A$ would be equivalent to $A \wedge B$. But that's intuitively false—indicative conditionals are weaker than their corresponding conjunctions. Witness: asserting $\mathit{if}\, B, A$ doesn't commit one to B, but asserting $A \wedge B$ does.

(iii) $\mathit{if}\, B, A \models B \supset A$. This is property holds iff indicative conditionals obey modus ponens. While there are interesting counterexamples to modus ponens for conditionals with conditional consequents (see McGee 1985), all we need here is that modus

[4]Strong centering is the principle that if its antecedent is true, then the truth value of the conditional goes by the truth value of its consequent:

(i) $\models A \supset (C \equiv \mathit{if}\, A, C)$.

[5]The standard resistance to this property is that a conditional like (18), whose consequent is not necessitated by its antecedent (relative to some background presuppositions), is false. Here's a quick argument against this thought. It seems that rationality demands that I should only accept a bet that what John said is true at 2:1 (or better) odds. But if (18) is false, and I can determine its falsity simply by reflecting on the open possibilities, then I shouldn't rationally accept such a bet, no matter what odds I'm offered.

ponens holds for all indicatives with simple (nonconditional) consequents—and that much seems unassailable.

(iv) $B \supset A \not\models \mathit{if}\, B, A$. Given (iii), if (iv) held, $\mathit{if}\, B, A$ would be equivalent to $B \supset A$—indicative conditionals would be equivalent to material conditionals. But this is false, as evidenced by the fact that $\mathit{if}\, B, A$ can be false even though B is false:

(19) If it's raining, then it's not raining.

(19) seems obviously false, and its falsity is not impeached by a lack of rain. Furthermore, the falsity of $\mathit{if}\, B, A$ does not entail that B is true:

(20) It's not the case that if it's raining, then it's not raining; and furthermore, it's not raining.

(20) has no feeling of contradiction, though it should if its indicative were equivalent to a material conditional.

Thus, the indicative conditional $\mathit{if}\, B, A$, like its logical remainder cousin $A - B$, is intermediate in strength between $A \wedge B$ and $B \supset A$. Consider the following diagram:

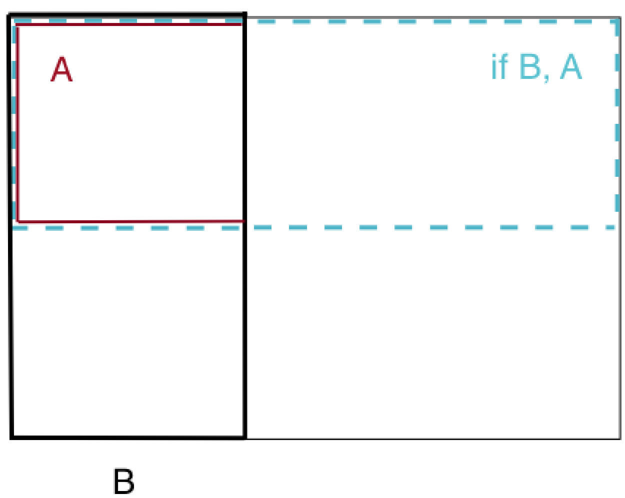

Figure 2

Like $A - B$, *if* B, A overlaps with $A \wedge B$, but extends beyond B, but not across the entire $\neg B$-region. Compare the partial truth table for *if* B, A:

B	A	*if* B, A
T	T	T
T	F	F
F	T	?
F	F	?

The challenge, as with logical remainders, is to figure out some principled recipe for determining just how far *if* B, A takes us beyond B—or, what considerations determine the truth value for *if* B, A when B is false. I think this is perhaps the most thorny question in the indicative conditionals literature, responsible for several of the motivations for non-propositional approaches.[6]

Most importantly for our concerns here is that logical remainders and indicative conditionals seem to share the above four properties and thus face an analogous challenge of determining their truth conditions. In the next section, I review my favored stab at this challenge for indicatives. In §5, I compare my theory with Yablo's approach to the challenge for logical remainders.

4 A semantics for indicatives

In this section, I'll sketch my favored semantics for indicative conditionals (see Khoo 2013a,b for additional motivation). The basic idea is not new—it goes back at least to Stalnaker 1968, 1975, 1980—but my twist on it brings it closer in line with Yablo's semantics for logical remainders. In what follows, I will simplify the presentation by leaving out several details—for instance that indicative conditionals are epistemic and context dependent and thus that their truth conditions are defined relative to a set of contextually relevant epistemically accessible worlds.[7] Doing so will allow me to talk of the truth conditions determined by the proposition expressed by an arbitrary indicative conditional directly. The basic semantics is as follows:

(21) *if* B, A is true at w iff A is true at the closest B-world to w.

[6] See in particular the discussion in Edgington 1995, Bennett 2003 and Khoo 2013b for my opinionated take on the situation.

[7] Readers interested in such complexities are invited to explore Khoo 2013a,b.

We flesh out the semantics by spelling out the nature of the closeness relation here:

- **Limit**: if B is true at w, then the closest B-world to w is w.
- **Similar**: if B is false at w, then the closest B-world to w is among the most relevantly similar worlds to w.

Limit ensures that strong centering and modus ponens hold, and hence that (i) $A \wedge B \models$ *if* B, A and (iii) *if* $B, A \models B \supset A$ hold. Now, an orthodox closest-worlds semantics like Stalnaker's will ensure (ii) and (iv) hold, since on such a semantics *if* B, A may be true even though B is false—the former is true since the closest B-world is an A-world—and $B \supset A$ may be true since B is false and yet *if* B, A be false since the closest B-world is a $\neg A$-world. Thus, we see that a closest worlds semantics has the resources to predict (ii) and (iv), however, it remains to be seen in what circumstances such a semantics predicts that *if* B, A is true when B is false—recall that this is just the challenge for determining the truth conditions of an arbitrary indicative conditional. In order to answer this challenge, we must say something substantive about **Similar**.

This is where my semantics departs a bit from the usual closest-world semantics for conditionals. **Similar** demands that certain respects of similarity between worlds matter for closeness and others do not. One way to model this is by way of a partition that fixes the relevant true propositions of w that must be held constant when finding the most similar worlds to w. Thus, let $\mathcal{Z}$ be a partition on W and $[w]_{\mathcal{Z}}$ be the cell of $\mathcal{Z}$ that w falls within.[8] Then, the most similar worlds to w given $\mathcal{Z}$ are among $[w]_{\mathcal{Z}}$—basically, this amounts to demanding that the most similar worlds to w are exactly like w where $\mathcal{Z}$ is concerned. Plugging this into **Similar** yields:

- **Similar***: if B is false at w, then the closest B-world to w is among $[w]_{\mathcal{Z}}$.

Now, if there are no B-worlds in $[w]_{\mathcal{Z}}$, then our semantics will crash: our truth conditions look for *the* closest B-world and **Similar*** will fail to provide *any*. Thus, we need a constraint on relevance partitions. I propose the following: a partition counts as a relevance partition for some conditional *if* B, A—that is, it fixes the matters of similarity that are

[8] A partition on a set of worlds W is a set of subsets of those worlds $\mathcal{Z}$ such that $\bigcap \mathcal{Z} = W$ and for which every $w \in W$ there is exactly one $Z \in \mathcal{Z}$ such that $w \in Z$. Less formally, $\mathcal{Z}$ is a set of boxes of worlds of W such that every world in W is in at least one box in $\mathcal{Z}$, and no world in any box of $\mathcal{Z}$ is in any other box in $\mathcal{Z}$.

relevant to its truth conditions—only if for each $\neg B$-world w, $B \cap [w]_{\mathcal{Z}} \neq \emptyset$ (basically, that $\mathcal{Z}$ crosscuts the $B/\neg B$ partition). Given this constraint, we ensure that our semantics will be non-defective.

However, notice that we do not ensure that **Similar*** will determine a unique B-world as closest—as long as there are two B-worlds in $[w]_{\mathcal{Z}}$, **Similar*** leaves it indeterminate which is the closest to w. It's a further complication how to handle this indeterminacy, and a question we won't enter into at this time (for discussion, see Khoo 2013a). This won't particularly matter for our purposes. For now, we'll just suppose the following (supervaluational) approach to the indeterminacy:

(22) *if* B, A is (relative to $\mathcal{Z}$)

a. true at w if A is true at all the candidates for being the closest B-world to w.

b. false at w if A is false at all the candidates for being the closest B-world to w.

c. undefined at w otherwise.

I defend the usefulness of such a semantics in Khoo 2013a,b, for reasons that I want to skip over for now. Instead, let's look at two more pieces of confirming evidence—that this semantics predicts the role intuitively played by indicative conditionals in Yablo's discussions of non-catastrophic presupposition failure and unexpected assertoric content.

Regarding non-catastrophic presupposition failure, recall that the goal is to predict that (1) is false and (2) unevaluable by predicting that (11) is false and (12) undefined:

(1) The King of France is sitting in that chair. (C) — False

(2) The King of France is bald. (B) — Undefined

(11) If there is a King of France, then he's sitting in that chair. (*if* K, C) — False

(12) If there is a King of France, then he's bald. (*if* K, B) — Undefined

Does our semantics predict that *if* K, C is actually false and *if* K, B actually undefined? Well, since there is actually no King of France, K is false and hence the value of both conditionals depends on what happens throughout $[\alpha]_{\mathcal{Z}}$, for the appropriate relevance partition $\mathcal{Z}$. A plausible thought is that the natural relevance partition for (11) is *how things with respect to the chair*, or perhaps just *whether the chair is occupied*, $\mathcal{Z}_1 = \{O, \neg O\}$

(where O = the chair is occupied).[9] Then, since $\alpha \in O$, every world in $[\alpha]_{\mathcal{Z}_1}$ is an O-world. Hence, every K-world in $[\alpha]_{\mathcal{Z}_1}$ is a $\neg C$-world. Hence, we predict that (11) is false. What about the relevance partition for (12)? Well, one possible partition is *how things stand with the bald people*—but if that were the relevance partition for (12), then it would be false as well. So, if we are to predict the undefinedness of *if* K, B then this must not be the partition. I suggest that the trivial partition $\{W\}$ is the relevance partition for (12), $\mathcal{Z}_2$. Then, since there are $K \wedge B$-worlds *and* $K \wedge \neg B$-worlds among W, there will be both kinds of worlds among $[\alpha]_{\mathcal{Z}_2}$ and hence we predict *if* K, B is undefined.

Turning now to our example of unexpected assertoric content, we want to predict that (17) is equivalent to (23), in a suitable context (thus confirming the role (17) plays as the interpolant in the argument (16)):

(17) If Joe is Smith's murderer, then Smith's murderer is insane. (*if* J, S)

(23) Joe is insane. (I)

The context I have in mind is one in which the relevance partition for (17) is *whether Joe is insane*, $\mathcal{I} = \{I, \neg I\}$. We aim to show that, given $\mathcal{I}$, *if* $J, S = I$. To do so, pick an arbitrary world w at which Joe is insane (so $w \in I$). Then, either J is true at w or not. Suppose J is true at w. Then, since I and J are true at w, S is true at w, so *if* J, S is true at w. Suppose J is false at w. Then, since I is true at w, by **Similar***, I is true at the closest J-worlds to w. But then S must be true at the closest J-worlds to w. So, *if* J, S is true at w. Thus, *if* J, S is true at all worlds in which I is true. Exactly analogous reasoning establishes that *if* J, S is false at all worlds in which I is false. Hence, given $\mathcal{I}$, *if* $J, S = I$.

Let's turn now to comparing our truth conditions for indicative conditionals with Yablo's truth conditions for remainders.

5 Comparisons with a semantics for remainders

Yablo gives us two (equivalent) routes to the truth conditions of logical remainders. For sake of space, I'll just review the first, comparing it with my truth conditions for indicative

[9] What features of the context suggest *this* relevance partition? One might be the sheer salience of the chair—for instance, it is being demonstrated (presumably), and talked about. However, I don't have any account of what features of context are relevant to determining the relevance partition for some conditional. I take it such an undertaking would lie on the (sometimes) hazy border between cognitive science and pragmatics.

conditionals:

(24) $A - B$ is

a. true at w iff A adds truth, but not falsity, to B in w.
b. false at w iff A adds falsity, but not truth, to B in w.
c. undefined at w otherwise.

What it is for a proposition to add truth/falsity to another at a world is defined in terms of truthmakers for the corresponding material conditional at that world:[10]

(25) a. A adds truth to B at w iff $B \supset A$ has a B-compatible truthmaker at w.
b. A adds falsity to B at w iff $B \supset \neg A$ has a B-compatible truthmaker at w.

As far as I can tell, for Yablo the truthmakers for a proposition depend on its subject matter—what the proposition is about—and what a proposition is about is something that floats free of its truth conditions (perhaps we should be talking about *sentences* rather than propositions here, I'm not so sure). Given a particular subject matter—understood as a division on W, $\mathcal{X}$[11]—we identify the set of propositions that are the truthmakers for some proposition A as follows:

(26) The truthmakers of A at w relative to $\mathcal{X} = \{P \in \mathcal{X} : w \in P : P \subseteq A\}$.

Hence, the truthmakers of A at w relative to $\mathcal{X}$ are those members of $\mathcal{X}$ that are true at w and entail A. If we put this together with our truth conditions for $A - B$ and the definitions of adds truth/falsity, we get:

(27) $A - B$ is (relative to $\mathcal{X}$)

a. true at w iff there is a $P \in \mathcal{X}$ that is true at w and such that $P \cap B \neq \emptyset$ and $P \subseteq B \supset A$; and there is no $P \in \mathcal{X}$ that is true at w and such that $P \cap B \neq \emptyset$ and $P \subseteq B \supset \neg A$.
b. false at w iff there is a $P \in \mathcal{X}$ that is true at w and such that $P \cap B \neq \emptyset$

[10] Things are slightly more complex: Yablo invokes the notion of a *targeted truthmaker* here, where a targeted truthmaker for $B \supset A$ is one that uses as much of B as it can—that is, by minimizing the extent to which $C \supset B$ is implied, where C ranges over the truth/false-makers of A. I won't discuss this additional feature here, but I would like to at some point!

[11] A division on W is a set of subsets of those worlds $\mathcal{X}$ such that $\bigcap \mathcal{X} = W$. The only difference between a division on W and a partition on W is that a division $\mathcal{X}$ does not require that each $w \in W$ be a member of exactly one $X \in \mathcal{X}$.

and $P \subseteq B \supset \neg A$; and there is no $P \in \mathcal{X}$ that is true at w and such that $P \cap B \neq \emptyset$ and $P \subseteq B \supset A$.

c. undefined at w otherwise.

Let's go through Yablo's semantics for $A - B$ to see what it predicts about our case of non-catastrophic presupposition failure and Donnellan's example of unexpected content. Regarding non-catastrophic presupposition failure, recall that the goal is to predict that (1) is false and (2) unevaluable by predicting that $C - K$ is false and $B - K$ undefined:

(1) The King of France is sitting in that chair. (C) False

(2) The King of France is bald. (B) Undefined

To determine whether $C - K$ is false requires that we check for K-compatible truthmakers for $K \supset C$ and $K \supset \neg C$, and find only ones for the latter. I'll assume that the relevant subject matter of (1) is a division that makes the fact that the chair is empty a truthmaker for $K \supset \neg C$. Furthermore, since this truthmaker is K-compatible, we have a truthmaker for $K \supset \neg C$. But we have no K-compatible truthmaker for $K \supset C$—notice that $\neg K$ is not one since it is not K-compatible. Therefore, we predict that $C - K$ is false. Turning to $B - K$, is there a K-compatible truthmaker for $K \supset \neg B$? One might think the fact that the bald people do not have among them the King of France such a truthmaker, but we'll assume that the relevant division knocks that out—thus, that (2) is about the King of France, *not* the bald people (this is the analogous assumption to my choice of relevance partition for *if* K, B above). Given this, there are no K-compatible truthmakers for either $K \supset B$ or $K \supset \neg B$ and hence we predict that $B - K$ is undefined.

Next, turn to (4)'s unexpected assertoric content:

(4) Smith's murderer is insane.

Recall that the theory aims to predict that (4) comes to have as its assertoric content that Joe is insane by the fact that $S - J$ interpolates between the semantic content of (4) and the background presupposition that Joe is Smith's murderer:

(28) a. Joe is Smith's murderer. (J)

b. Joe is insane. ($S - J$)

c. Smith's murderer is insane. (S)

Thus, we want to ensure that our semantics predicts that $S-J$ in this context is equivalent to that Joe is insane ($S-J=I$). A natural assumption in this context is that the subject matter is *whether Joe is insane*, $\mathcal{I}=\{I,\neg I\}$. Let w be an arbitrary world in which Joe is insane (so $w\in I$). Now, since $I\subseteq J\supset S$, but $I\not\subseteq J\supset S$, it follows that $S-J$ is true at w. Thus, $S-J$ is true at every world at which I is true. And analogous reasoning establishes that $S-J$ is false at all worlds in which I is false. Hence, given $\mathcal{I}$, $S-J=I$.

So far, we've seen that both my semantics for *if* B, A and Yablo's semantics for $A-B$ predict plausible results for the role played by remainders/indicatives in the theory of non-catastrophic presupposition failure and unexpected assertoric content. But the truth conditions each theory assigns respectively to *if* B, A and $A-B$ ensure that the two are not equivalent. To begin with, notice that Yablo's truth conditions for $A-B$ are defined relative to divisions, while mine are defined relative to partitions. My semantics relies on finding *the* cell of the relevance partition w falls within to fix the respects of similarity that matter, whereas Yablo's semantics allows for multiple such "cells" to play an analogous role in his semantics. This difference gives rise to the potential for non-equivalence. My truth value for *if* B, A at w depends only on whether $B\supset A$ is true throughout the cell of the partition $\mathcal{Z}$ that w falls within. Yablo's truth value for $A-B$ at w depends on whether $B\supset A/B\supset\neg A$ is true throughout potentially many propositions in the division $\mathcal{X}$ that are true at w. This is the crucial difference between the two semantics, and it's not hard to set up a case in which *if* B, A is true on my semantics and $A-B$ false on Yablo's (by simply choosing a partition for the former that bears no resemblance to the division for the latter).[12] However, even if the relevance partition for *if* B, A is a subset of the division relative to which $A-B$ is computed, it may turn out that *if* B, A is true (or false) and $A-B$ undefined (by adding additional truthmakers for the material conditionals, we add more chances for $A-B$ to be undefined). Only when the relevance partition for *if* B, A is identical to the division relative to $A-B$ is computed are the two predicted to be equivalent.[13]

Thus, we face a fork in the road: either $A-B$ and *if* B, A are equivalent, in which case both my and Yablo's truth conditions are after the same thing but offer conflicting accounts

[12]A related difference is my stipulation that relevance partitions for conditionals crosscut those conditionals' antecedents. The problem this avoids is sidestepped in Yablo's semantics by the stipulation that the truthmakers for $B\supset A/B\supset\neg A$ be B-compatible—and hence, given his definition of undefinedness for $A-B$, failures of B-compatibility for all such truthmakers are an additional route to undefinedness.

[13]This is so because, where $\mathcal{Z}$ is the relevance partition for *if* B, A and the relevant division for determining $A-B$, the truth of both at w depends just on whether $B\supset A$ is entailed by the $P\in\mathcal{Z}$ such that $w\in P$.

of it, or $A - B$ and *if* B, A are not equivalent, in which case our proposed truth conditions might be offering compatible characterizations of different things. The discussion in §1–4 suggests equivalence, but the case is far from closed. Indeed, deciding this matter involves an additional epistemic hurdle, for even if they are equivalent, any evidence favoring one semantics over the other might equally well be taken by the defender of the other semantics as grounds for non-equivalence. Thus, we must proceed with caution. Nonetheless, in this paper I hope to have at least provided some evidence that logical remainders and indicative conditionals share enough similarities to take seriously the thought that they might be the same thing. They seem to play the same theoretical role in explaining non-catastrophic presupposition failure, cheap ontological arguments, and cases of unexpected assertoric content (§1–2). Furthermore, both are subject to a similar puzzle about their truth conditions (§3). And finally, in many (but not all) cases, the theories I and Yablo provide of each will ensure that they are equivalent (§4–5). I hope this paper has added some additional resources for how to think about remainders and indicative conditionals.[14]

References

Bennett, Jonathan. 2003. *A Philosophical Guide to Conditionals*. Oxford: Oxford University Press.

Donnellan, Keith. 1966. Reference and Definite Descriptions. *The Philosophical Review*, **75**(3), 281–304.

Edgington, Dorothy. 1995. On Conditionals. *Mind*, **104**, 235–329.

Gibbard, Allan. 1981. Two Recent Theories of Conditionals. *Pages 211–247 of:* Harper, William L., Stalnaker, Robert, & Pearce, Glenn (eds), *Ifs*. Dordrecht: Reidel.

Karttunen, Lauri. 1973. Presuppositions of Compound Sentences. *Linguistic Inquiry*, **4**(2), 169–193.

Khoo, Justin. 2013a. *Probability of Conditionals in Context*. ms.

Khoo, Justin. 2013b. *Propositionalism, Indeterminacy, and Triviality*. ms.

McGee, Vann. 1985. A Counterexample to Modus Ponens. *The Journal of Philosophy*, **82**(9), 462–471.

[14] For helpful discussion, I would like to thank Pamela Corcoran, Ben George, Mark Maxwell, Aaron Norby, Zoltán Szabó, and Jonathan Vertannen.

Rothschild, Daniel. 2011. Do Indicative Conditionals Express Propositions? *Nous*.

Stalnaker, Robert. 1968. A Theory of Conditionals. *Pages 98–112 of:* Rescher, N. (ed), *Studies in Logical Theory*. Oxford: Oxford University Press.

Stalnaker, Robert. 1975. Indicative Conditionals. *Philosophia*, **5**, 269–86.

Stalnaker, Robert. 1980. A Defense of Conditional Excluded Middle. *In:* Harper, William L., Pearce, Glenn, & Stalnaker, Robert (eds), *Ifs*. Dordrecht: Reidel.

Strawson, Peter. 1954. A Reply to Mr. Sellars. *Philosophical Review*, **63**, 216–231.

Yablo, Stephen. 2006. Non-Catastrophic Presupposition Failure. *In:* Byrne, Alex, & Thomson, Judith Jarvis (eds), *Content and Modality: Themes from the Philosophy of Robert Stalnaker*. Oxford: Oxford University Press.

Yablo, Stephen. 2013. *Aboutness*. Princeton University Press.